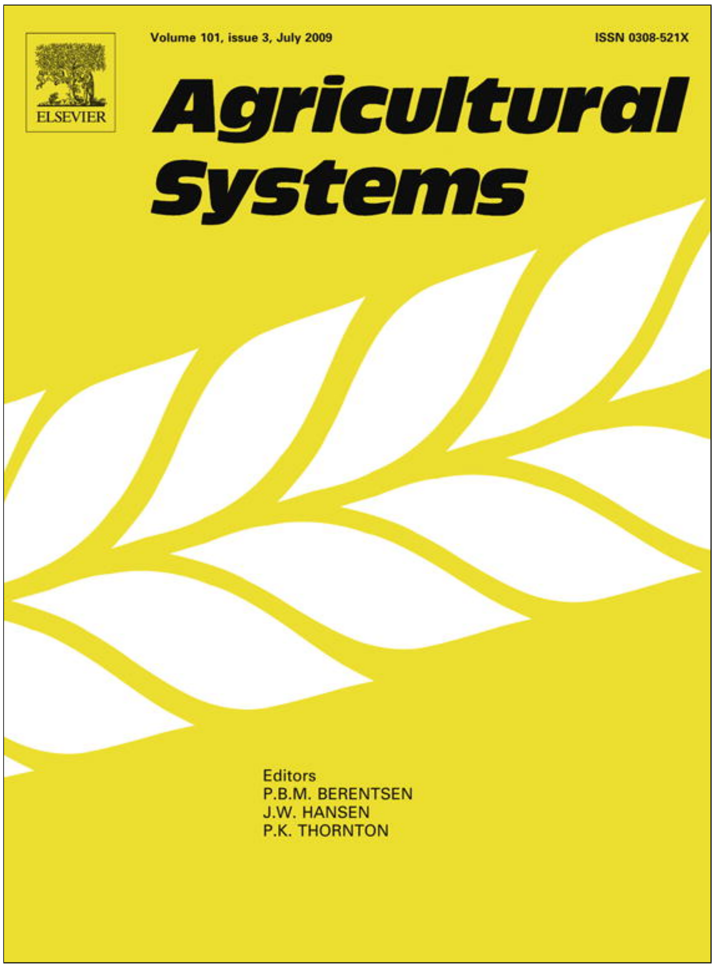
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A quantitative framework to analyse cooperation between rural households

M. Gerichhausen [a,b,*], E.D. Berkhout [b,1], H.J.M. Hamers [a], V.M. Manyong [c]

[a] Tilburg University, CentER and Department of Econometric and Operations Research, P.O. Box 90153, 5000 LE, Tilburg, The Netherlands
[b] International Institute of Tropical Agriculture (IITA), PMB 3112, Kano, Nigeria
[c] International Institute of Tropical Agriculture (IITA), P.O. Box 34441, Dar es-Salaam, Tanzania

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ABSTRACT

Different types of cooperative agreements between smallholders continue to play an important role in rural areas in developing countries. While some empirical studies examine the conditions catalysing the successful formation of cooperatives, quantifications of the net benefits, i.e., difference between revenues and costs, of cooperation and how farmers divide these net benefits are scarce. Therefore, we develop a quantitative framework to analyse and allocate net benefits in a cooperative production agreement. The framework allows for cooperative exchange of several types of resources and the production of multiple products.

Linear programming provides insight into optimal production levels, both for individual and cooperating farmers, and gives optimal revenue levels. A transaction cost function is used to account for costs of cooperation, such as meeting costs, moral hazard and free ridership of labour use and the risks of farmers defaulting from the agreement. Transaction costs are likely to increase with the number of households participating, the total cropping area and the heterogeneity of resources of the cooperating farmers. Therefore, we introduce a measure of heterogeneity in the resources for each cooperative. Finally, cooperative game theory is used to generate fair divisions of the net benefits in a cooperative.

This framework may be used to give additional explanations to the findings in empirical studies on cooperatives. We illustrate this with an empirical example from northern Nigeria. It is found that cooperation between farmers sharing complementary resources gives the highest revenues. Next, we illustrate the effects of two different transaction cost functions. For reasonable assumptions on these functions, cooperation remains economically attractive. Nevertheless, larger and more diverse coalitions are not always the most beneficial, while the returns in some small coalitions are negative, possibly impeding the formation of cooperatives in some locations.

1. Introduction

In Sub Saharan Africa approximately 64% of the population lives in rural areas (FAOSTAT, 2007), earning their income primarily from farming and related activities. Most of these farmers use mainly household labour on relatively small, subsistence-oriented farms. Their cash income is limited and is either derived from limited sales of crops or from limited off-farm labour opportunities. Additional income is used to purchase food and other primary necessities of life. A major impediment to the economic growth of these smallholder farmers is weak rural infrastructure (e.g. World Bank, 2008a), such as bad roads leading to poor access to input and output markets.

As a result of poor rural infrastructure, lack of credit markets, risk and uncertainty in production and prices, and information asymmetry, many rural households do not participate in one or more input and output markets. If the disutility generated by a trade in the market, through transaction costs, exceeds the utility gain to a farmer, the market fails for this farmer (de Janvry et al., 1991). Farmers will then avoid trade, and will therefore not use their resources efficiently. Moreover, smallholder farmers are heterogeneous with respect to their resources as well as participation in markets (e.g. Ruben and Pender, 2004). For example, some farmers have excess land, given their labour and capital resources, while other farmers may have a surplus of other resources. Hence, it is expected that policy measures aimed at facilitating trade of these excess resources will improve the well being of smallholder farmers.

On the other hand, partially as a result of these market failures, many farmers have engaged in cooperative production agreements, such as casual labour exchange. While such agreements continue to play an important role for many farmers in developing

* Corresponding author. Address: Tilburg University, CentER and Department of Econometric and Operations Research, P.O. Box 90153, 5000 LE, Tilburg, The Netherlands. Tel.: +31 13 4662478.
E-mail address: m.gerichhausen@uvt.nl (M. Gerichhausen).
[1] Present address: Wageningen University and Research Center (WUR), Development Economics Group, Hollandseweg 1, 6706 KN, Wageningen, The Netherlands.

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countries (e.g. Worby, 1995; Gilligan, 2002; Tu and Bulte, 2007; Bernard et al., 2008a), few quantifications have been made of the costs and benefits in economic studies, and if and how additional gains are divided.

Therefore, we develop a quantitative framework to model cooperative agreements, based on sharing resources, amongst smallholders in developing countries. The aim of this framework is threefold. The first aim is to optimise the farm plan of a cooperative, for which we use linear programming (LP). This farm plan of the cooperative can thereby differ from aggregated individual farm plans, based on complementarities in resources. The second aim is to calculate the net benefits of a cooperative. Therefore, we take the difference between the revenues of a cooperative, resulting from the LP-model, and the transaction cost of cooperation. The third aim is to demonstrate how the net benefits of the cooperative can be divided amongst the members in a fair way, by using cooperative game theory. To address these aims, the framework developed consists of four components.

Firstly, we develop a farm household model using linear programming (LP), which is used to determine optimal farm plans for each possible combination of farmers. This model is based on the commonly applied agricultural farm household model, e.g. described by Schweigman (1985) and Hazell and Norton (1986). An extensive body of research uses such linear or non-linear models to analyse farm household decisions. Hazell and Norton (1986) provide an overview, while more recent applications include, Abdoulaye and Sanders (2006), Dorward (2006), and Woelcke (2006). These models represent the main decisions in a farm household, namely, production, market and consumption decisions. The constraints reflect the major resources used in farming: land, labour, and capital. The solution of the model leads to an optimal farm plan.

Secondly, we model a farm cooperative in which farmers group their resources, i.e., land, labour and capital, and jointly make decisions based on the aggregate resources, after which they divide the gains of the joint production in a fair way. Note that a farm cooperative is different from a collective farm, which is operated and owned by a group of people, i.e., like the Kibbutz in Israel. This is contrary to our approach, where farmers remain independent. To model such a farm cooperative we develop a cooperative farm household game, which combines two drivers that play an important role in forming cooperatives: the revenues and the transaction costs. Revenues of cooperatives are calculated using the farm household model. On the assumption that cooperation leads to more efficient use of resources, total revenues from farm production in a cooperative are expected to increase. Without including the transaction costs of cooperation, the cooperative farm household game with only revenues provides an upper bound or benchmark on the potential gains achieved by cooperating.

However, the size of many cooperatives is assumed to be suboptimal due to the presence of transaction costs resulting from holding meetings, monitoring participants, moral hazard and free ridership in labour use, increased travel costs and distrust between differently endowed households (e.g. World Bank, 2008b). Furthermore, this includes costs induced by the risk that certain members may default from the cooperative agreement. Therefore, it is likely that transaction costs play a critical role in the formation of cooperatives. Hence, by including a transaction cost function we are able to determine when cooperation is economically rational and when it is not. We assume that the transaction costs depend on the number of households in the cooperative, the heterogeneity of their resources, and the cropping area in the cooperative. These components are included through a linear or exponential transaction cost function.

Thirdly, if the resulting net benefits in a cooperative are positive, a problem arises of how to divide these benefits amongst the cooperating farmers. Most forms of cooperative labour use, as observed in various locations (e.g. Worby, 1995), are simple: standard man-hours of labour are usually exchanged one-to-one. Mostly similarly endowed households cooperate. These households face relatively similar shadow prices, which make it easy to agree on such a one-to-one exchange. This could mean that cooperation between heterogeneous farmers is hampered by difficulties in valuing and exchanging complementary resources. Hence, the development of fair rules to divide costs and benefits in more complicated cooperative agreements is likely to facilitate cooperation between farmers with complementary resources.

Clearly, a farmer is only inclined to join a cooperative if the expected return is higher than that achieved when producing alone. Moreover, a farmer is likely to remain part of the cooperative in the long run, if his/her return is in proportion to his/her contribution, and if there is no distrust about the fairness of the division. Therefore, we use the Shapley value (Shapley, 1953), which is a division rule from cooperative game theory. Cooperative game theory is a mathematical framework to analyse cooperation, and its main focus lies with the study of divisions of joint net benefits in cooperatives. Empirical applications of cooperative game theory in agriculture and land use include Suzuki and Nakayama (1976), Aadland and Kolpin (1998) and Lejano and Davos (1999), while applications in a rural African setting are scarce.

Finally, some empirical evidence suggests that farmers similar in endowments or, in general, farmers in villages characterised by homogeneity in endowments, are more likely to cooperate (Gilligan, 2002; Bernard et al., 2008a). An important explanation could be that distrust amongst similarly endowed neighbouring farmers is smallest (Tu and Bulte, 2007). The reason that many observed cooperatives are composed of socially homogenous types of farmers is sometimes described as the 'middling effect' (e.g. Thorp et al., 2005). Hereby the poorest farmers are excluded since their contribution to a cooperative is smallest; while the richer farmers well integrated in local markets, frequently have no need to join a cooperative. As a result, cooperatives tend to form mostly amongst averagely endowed farmers. Furthermore, local norms and values related to mutual assistance appear important for the successful development of different types of cooperatives (e.g. Bernard et al., 2008b). Finally, in certain types of cooperation, such as the joint marketing and cultivation of, e.g. a high-value crop, a homogenous composition of the cooperative may actually be beneficial. However, our hypothesis is that the revenues of cooperation in production are positively related with the degree of heterogeneity of resources in a cooperative, because complementary resources lead to higher revenues.

To capture this relationship, a measure of heterogeneity in resources is required, as acknowledged in many studies. So far no consensus has emerged which measure to use. For example Gilligan (2002) includes the variation in land size, Gebremedhin et al. (2004) use diversity in oxen ownership, and Bernard et al. (2008a) include a measure of caste and ethnic diversity, while Bardhan (2000) uses the GINI-coefficient. The measure introduced in this paper combines the upland to labour and lowland to labour ratios. The measure is used to determine the relation between the revenues of coalitions and the heterogeneity of resources. Moreover, we assume that transaction costs depend on the heterogeneity in resources in a cooperative.

There is a widespread literature on the different forms in which cooperatives come in place in developing countries (e.g. Thorp et al., 2005; Uphoff and Wijayaratna, 2000; Bernard et al., 2008a). The framework developed in this paper primarily expands on cooperative agreements in crop production, mainly through the cooperative exchange of labour as observed in various smallholder environments. While such labour exchange teams have received some attention in literature (Moore, 1975; Worby, 1995; Gilligan, 2002), only few analyse such practice from an economic perspective. Labour exchange was expected to disappear with further development of markets (e.g. Moore, 1975) but still exists today, even in areas well integrated into markets and dominated by production of export commodities such as cotton (e.g. Worby, 1995). For Burkina Faso, Bernard et al. (2008a) find that 76%, of nearly 300 community-oriented village organizations surveyed, maintain collective fields, exchange labour and/or maintain a common cereal bank.

In the case of labour exchange, the main hypothesis put forward for the persistence of such cooperation in areas with well-developed markets relates to economies of scale in teamwork. This occurs when certain labour-demanding activities such as transplanting rice and picking cotton need to be carried out quickly and precisely timed quite precisely. Furthermore, the knowledge that a farmer has access to a relatively large supply of labour by participating in a labour cooperative reduces uncertainties in seeking wage labour. Moreover, for cash-constrained households exchanging labour during the cropping season without actually paying a wage is beneficial. Finally, the reciprocity of labour exchange is hypothesized to reduce loss of labour quality due to moral hazard and free ridership, while this loss could be considerably higher when hiring wage labour.

To analyse the underlying decision-making process, Gilligan (2002) develops a theoretical model based on the production of a single crop, and demonstrates that positive returns to team-labour are necessary in a single-crop environment. However, this finding might not be transferable to a multiple-crop environment, since a farmer could adjust relative crop areas based on the certainty of accessing labour through a cooperative agreement. In that case, if increased profits from adjusting crop areas outweigh possible negative returns to teamwork and/or potential losses resulting from moral hazard and free ridership, cooperation could still be preferred.

In addition to cooperation in crop production, various other formal and informal cooperative agreements are observed. For example, farmers frequently cooperate in associations to market products and purchase inputs in order to increase negotiation power with traders or rural services providers (Poulton et al., 2005), mostly in the case of producing high-value commodities. Moreover, in several countries cooperatives have formed with the aim of providing financial services (e.g. Cuevas and Fischer, 2006). Sometimes such groups are backed by a formal credit organisation with the main benefit to the latter being the low default rates resulting from own-peer selection in such groups (e.g. World Bank, 2008b). The management of natural and water resources, i.e., 'tragedy of the commons' type of problems, has been addressed extensively both in theoretical (e.g. de Janvry et al., 1998) as well as in empirical literature. For example pastoralists in East-Africa, negotiate access for their flocks to fields of neighbouring pastoralists and vice versa (Boone et al., 2005). Moreover, cooperative agreements in sharing irrigation resources are well documented (e.g. Uphoff and Wijayaratna, 2000; Bardhan, 2000). Many of these examples are capital and/or labour-intensive, such as building dikes in managing water resources, and are more effectively implemented jointly by larger groups (Knox and Meinzen-Dick, 1999). Finally there is considerable evidence that farmers cooperate informally in insuring (idiosyncratic) risks (e.g. Ravallion and Chaudhuri, 1997).

So far, empirical evidence suggests that farm cooperatives are being formed amongst similarly and/or averagely endowed farmers (e.g. Thorp et al., 2005). Other evidence suggests that the success of farm cooperatives depends on the population density of the area, whereby cooperation is less attractive in very low or very densely populated areas (Gebremedhin et al., 2004). Finally both benefits and costs increase with the size of the cooperative and by consequence it is found in Kenya that the best performing cooperatives are averagely sized (Place et al., 2004). Hence, while there is some empirical evidence on the main conditions catalysing the successful formation of farm cooperatives, the exact underlying mechanism of costs and benefits to farmers engaging in cooperation largely remains a black box.

The framework presented in this paper allows for a quantification of costs and benefits of cooperation to heterogeneously endowed farmers. Furthermore, while it does allow for cooperation based on sharing one resource, our framework is suitable to analyse cooperation based on multiple resources and/or analysing the effects of cooperation in a multiple-crop environment.

The remainder of this paper is organised as follows. In Section 2, we introduce the framework of analysis by combining methods from linear programming and game theory. Thereafter, in Section 3, we illustrate the applicability of the framework by presenting an example of rural households in northern Nigeria. Finally, in the discussion in Section 4, we suggest further strategies for theoretical and empirical research.

2. The quantitative framework

In this section, we introduce the quantitative framework that provides a base for analysing cooperative farm decisions in developing countries, which is applied to a group of farm households in northern Nigeria in Section 3. The framework combines two mathematical disciplines, linear programming and cooperative game theory. It is used in modelling decision-making of a farm cooperative. The advantage of using cooperative game theory in combination with LP is that an optimal farm plan of a group of farmers can be determined. Moreover, fair division rules from cooperative game theory can be used to divide the gain obtained in the farm cooperative. In this section, we first describe the LP model, used in modelling both individual farmer's decisions and cooperative decisions. Thereafter, we introduce the concept of cooperative games and develop a cooperative farm household game. Next, various solution concepts from cooperative game theory are briefly described. Finally to conveniently compare different cooperatives, we define the measure of resource heterogeneity in a cooperative.

2.1. The LP model

The mathematical formulation of an LP problem is

$$\max\{c^T x \mid Ax \leqslant b;\ x \geqslant 0\}, \tag{1}$$

where $x \in \mathbb{R}^k$ represents the decision variables, $c \in \mathbb{R}^k$ represents the objective function coefficients, $b \in \mathbb{R}^m$ is the resource bundle, and $A \in \mathbb{R}^{k*m}$ is the production matrix. In applications in agricultural economics such as ours, c commonly reflects exogenous market prices of commodities, b reflects the different resources of a farmer and the production matrix A relates use of labour, land and chemical inputs to yield outcomes.

Most LP models in rural settings in developing countries, as will be ours in the illustrative case presented in Section 3, are inspired by the basic farm household model as presented in Schweigman (1985, pp. 18–30) and Hazell and Norton (1986). The first element of an LP model consists of the decision variables. The decisions a farmer has to take are manifold, though one can generally group these into decisions on production, consumption and marketing (Table 1).

The production decisions of a farmer can be expressed by the assignment of land to different cropping systems and is called a farm plan. The farm plan defines total production quantities of different crops, as well as describes which production methods to use and when to apply labour, fertiliser, etc. Observe that leaving land

Table 1
Decision variables in farm household model.

Type of decision variables	Description decisions
Production decisions (yearly)	Assignment of area to cropping systems
Consumption decisions (monthly)	Consumption of different crops
Market decisions (monthly)	Selling/purchasing of different crops
	Buying fertiliser
	Hiring labour/out hiring labour
	Taking/paying off a loan

Table 2
Overview constraint types in farm household model.

Type of constraint*	Resource parameter
Land	*Available area*
Common fields (1)	Common
Fadama fields (2)	Fadama
Labour (monthly) (3)	Available labour
Storage balances (monthly calculation of quantity crops in store) (4–8)	Initial storage
Capital balances (monthly availability of money) (9–10)	Off-farm income and other expenditures, initial capital
Subsistence constraints (11–12)	Minimal nutritional needs
Loan constraints	
Paying back loan (13)	Maximum amount available loan
Maximum loan (14)	
Time which is spent on wage labour (monthly) (15)	Maximum hours available to spend on wage labour

* The numbers in brackets behind the constraint types refer to the inequalities in the Appendix.

fallow is also an option. These different combinations are accounted for through the introduction of different cropping systems in the production matrix. Each cropping system describes a yield or production level for one or more crops in the system, in relation to predefined levels of labour use, time of labour application and fertiliser application. For example, different cropping systems are included in the production matrix to account for different possible levels of fertilizer input in rice. Hence, a farmer can assign a piece of land at which rice is cultivated with high use of fertilizer, low levels of fertilizer or both.

The consumption decisions depend on the nutritional value of crops, the food habits and the availability of crops. The farmer needs to decide how much his family consumes of each crop in each month.

The most important market decisions are on trading crops, buying fertiliser, hiring (out) labour, and contracting or repaying a loan. Note that all decision variables are related to each other. For example, decisions on selling and buying of a certain crop are related to the production of this crop and its chosen consumption level.

The second element of an LP model is the objective or utility function. The farmers' objective in the model is to maximise the gross margin of the crop production. To calculate the gross margin, the costs of hired labour and fertiliser use in production are subtracted from the total production valued at market prices. The value of production depends on the chosen farm plan, the yields and the prices of the different crops. The cost of hired labour depends on the total number of hired labour hours and the wage rate, while the cost of the fertiliser depends on the farm plan, the required fertiliser inputs for each cropping system and the fertiliser prices.

This objective has been used frequently in applications of farm household models in SSA (Hazell and Norton, 1986). Many authors (Upton, 1996; Abdoulaye and Sanders, 2006; Woelcke, 2006) claim that monetary objectives, like gross margin, profit, income, net revenue optimisation are suitable if a subsistence constraint is included in the model to guarantee sufficient consumption in the household. We show later that in our model this constraint is included. Further, note that when modelling cooperation, dividing cash amongst farmers is easily interpretable.

It is likely that heterogeneity in utility functions, through e.g. consumption and risk preferences, as well as differences in market transaction costs render the assumption of profit maximisation infeasible for some farmers. However, since the primary focus of this paper is to introduce a framework in which to analyse the gains and benefits from cooperation, we do not further address such differences and leave to this to further development and/or detailed empirical applications.

The third element of an LP model consists of the constraints. The constraints need to properly reflect the actual farm household situation. Table 2 gives an overview of the constraint types of the model.

First, we introduce a land constraint. The total area used for the different cropping systems cannot exceed the available land. To make the model specific to the region of northern Nigeria considered in Section 3, we include an extra restriction for the use of fadama area (low lands) for cropping systems that include crops with high demand for water, such as rice and sugarcane.

Second, we incorporate monthly constraints for labour use, whereby the labour demand incurred by the chosen farm plan cannot exceed the available labour (including hired labour and excluding labour hired out) supply. These constraints are included on a monthly basis because the requirements fluctuate during the year. During the weeding and harvesting period the labour requirements are high, while cropping activities incur no labour requirements outside the growing season.

Furthermore, subsistence consumption requirements are met by incorporating two constraints for the minimum nutritional intake of energy and protein. These constraints guarantee that sufficient amounts of energy and protein are produced and purchased to meet minimal household requirements during the full year starting from the harvest date. Further, a restriction is introduced for perishable crops, which cannot be stored for a long period (i.e., vegetables) and two constraints are incorporated to include the crop leftover of the previous cropping season as initial storage. The loan taken during a year should be paid back before the end of the year.

We made assumptions on the functioning of the different input markets as follows. With respect to capital we assume that there is a maximum amount of money which can be borrowed. Moreover, to reflect the imperfections on the labour market, we included a constraint to set a maximum to the monthly amount of time the farmer is able to work on other farms, earning additional income against the local wage rate. Finally, we assumed that land is not commonly traded.

A constraint further initialises capital levels at the start of the growing season. While, finally, we include monthly storage and capital balances. The capital balance depends on the income and expenditures. Each month the actual capital level changes, based on: changes in loans; income and expenditures of trading in crops; expenses on required fertiliser and hired labour; income from labour hired out; off-farm income and other expenditures.

2.2. Cooperative farm household game

In cooperative game theory economic agents are called *players*. Let N be a finite set of players, in our framework farm households, and let 2^N denote the collection of all subsets of N, which are called *coalitions*. The coalition in which all players are included, N, is called the *grand coalition*.

A *cooperative game* is the pair (N,v), where $N=\{1, 2, \ldots, n\}$ is the set of players, and $v : 2^N \to \mathbb{R}$ is a map assigning to each coalition $S \in 2^N$ a real number, such that $v(\emptyset)=0$. The function v is called the characteristic function of the game and $v(S)$ is called the *value of coalition S*.

We quantify the net benefits of cooperation by introducing a cooperative farm household game. Let (N,r) be a cooperative game representing the revenues of the coalitions and let (N,c) be the cooperative game representing the costs of the coalitions. Then the net benefits of coalitions are expressed in a cooperative farm household game (N,v), where $v(S) = r(S) - c(S)$ for all coalitions S. The revenue and cost game are subsequently described in the next sections.

Furthermore, to investigate the relation between the revenues of cooperation and the level of heterogeneity in resources of farmers in the coalition, we introduce the following definition of the increment in net benefits. The increment in net benefits $\Delta(S)$ of coalition S is the difference between the value (net benefits) of coalition S, $v(S)$, and the sum of individual values (net benefits) of farmers in coalition S, i.e.,

$$\Delta(S) = v(S) - \sum_{i \in S} v(\{i\}). \quad (2)$$

Hence in the expression $\sum_{i \in S} v(\{i\})$ in Eq. (2) represents the sum of net benefits farmers can earn when operating alone.

2.2.1. Cooperative game for revenues

To calculate the revenues we use a linear production (LP) game (Owen, 1975), which combines LP models to create cooperative games. Let $c, x \in \mathbb{R}^k$ and matrix $A \in \mathbb{R}^{k*m}$ as in the LP model (Eq. (1)) and $b_i \in \mathbb{R}^m$ denote a resource vector for each individual player $i \in N$. The *linear production game* (N,r) is a cooperative game in which the characteristic function is defined as:

$$r(S) = \max\{c^T x \mid Ax \leqslant b(S),\ x \geqslant 0\}, \quad (3)$$

where $b(S) = \sum_{i \in S} b_i$ is the resource bundle owned by coalition S. Hence, the LP model determines the value (revenues) of coalition S.

The ingredients needed for an LP game, are the players, the objective function, the production matrix and the resource bundles. Three main characteristics of LP games are that the production matrix and the objective function are equal for every player, i.e., the households from a certain village, while the resource bundle can differ. These characteristics are assumed to hold in our case. First, we assume production technologies to be homogenous at village level, and that by consequence every farmer takes decisions based on the same production matrix, further ignoring potential local differences in climate, soil fertility and managerial quality. Furthermore we assume that farmers face similar exogenous prices and are homogenous in the used objective function. Second, as observed before, farm households are heterogeneous in resources and each player has a different resource bundle.

2.2.2. Cooperative game for transaction costs

To define the cooperative game (N,c) for the transaction costs of cooperation, we first consider the several types of costs that are involved in engaging into a cooperative production agreement, such as meetings, monitoring and supervision costs. Furthermore, note that besides these costs, transaction costs represent also a risk that farmers encounter when entering in cooperatives.

First, there are costs of meeting(s) before the cropping season in which joint decisions are made on the farm plan. Moreover, there are costs of meetings during the farm season for coordinating the actual activities and planning the day-to-day use of labour. We hypothesize that these costs depend on the number of households in the coalition and the total area of land managed in the coalition. The time to agree upon decisions are likely to increase both with the number people involved and with the total farm land of the coalition.

Secondly, throughout the cropping season there are supervision costs, mainly to reduce the consequences of moral hazard and free ridership in labour supply. In general, farmers are believed to work more effectively on their own land (e.g. Gilligan, 2002). Hence standard units of wage or hired labour are of a lower quality than household labour. On the other hand the reciprocal nature of sharing labour in labour teams and the associated social control is hypothesized to reduce the effects of moral hazard (e.g. Worby, 1995). Furthermore as farmers remain supervisor of their own fields, while implementing the joined production decisions, earlier agreed upon, the direct supervision costs are likely to be similar to individual production plans.

However, these preceding arguments hold mostly if cooperating farmers are similar in resources, suggested by some of the empirical evidence on labour exchange. If farmers are heterogeneous in resources, such as in our case, the actual exchanges of labour are less reciprocal. In such a case the moral hazard and free rider problems are expected to be higher and by consequence the supervision costs as well. Hence, we assume supervision and management costs during the cropping season to increase for coalitions in which the farmers are dissimilar with respect to resources. Furthermore, the risk that farmers default from the cooperative agreement is likely to increase in cooperatives with a heterogeneous composition.

Thirdly, costs are involved to travel to and from the fields. These costs depend on the distance to the farms and the number of people travelling. The distance to farms is likely to increase with the total area under cultivation, while the number of people travelling depends on the degree to which farmers are complementary in resources. Thereby land scarce farmers relatively more often work with labour-scarce farm households.

Finally, in a labour team the owner of the fields receiving labour is expected to provide food and drinks (e.g. noted by Worby, 1995), which increase with the size of the cooperative.

By the preceding discussion of cost components we assume that the transaction costs ($c(S)$) of forming and implementing a coalition S depend on the number of households in the coalition ($s = |S|$), the total area managed by the coalition ($a = \text{Area}(S)$) and the degree of heterogeneity in resources of the coalition ($h = H_S$), whereby $h = H_S$ reflects the degree of heterogeneity of the coalition S (see Section 2.4). Hence, the function describing transaction costs of a coalition, $tc(s,a,h)$ increases in its arguments: $df/ds > 0$, $df/da > 0$, $df/dh > 0$. In the empirical example (Section 3), we analyse the effects for several shapes of this function. Two forms are considered, a linear function:

$$tc_1(s,a,h) = \alpha_1 s + \beta_1 a + \chi_1 h \quad (4)$$

and an exponential function:

$$tc_2(s,a,h) = \alpha_2 s^2 + a^{\beta_2} * h^{\chi_2}. \quad (5)$$

Finally, we denote the transaction cost incurred by coalition S, with $|S| > 1$ as: $c(S) = tc(s,a,h)$. Note that transaction costs of cooperation are zero for one-player coalitions, i.e., $c(\{i\}) = 0$ for all $i \in N$.

2.3. Solution concepts from cooperative game theory

A key subject in cooperative game theory is how the value of the grand coalition, i.e., formed by all players or farm households, should be divided amongst the players, such that all players have an incentive to cooperate. A frequently used division rule (or solution concept) in applications in literature is the Shapley value (e.g. van den Nouweland et al., 1996; Land and Gefeller, 2000), introduced by Shapley (1953). The reason that the Shapley value is frequently used in literature is twofold. First, the Shapley value is based on the marginal contribution of a player if he joins some coalition. This marginal contribution reflects a measure of importance of the player in the game. Second, the Shapley value satisfies many desirable properties. We mention here two of these properties: symmetry and monotonicity. Symmetry implies that if two players have a similar contribution in the game, the solution concept assigns the same value to both. Monotonicity implies that if one player contributes more than a second player in the game, the value assigned to the first player is higher.

In cooperative game theory many solution concepts exists, e.g. the nucleolus (Schmeidler, 1969), the compromise value (Tijs, 1981) that have different properties than the Shapley value. Depending on the scope and setting of the problem one solution concept is more appropriate than the other. In our case, we have chosen the Shapley value, because of the two before mentioned reasons, which fits best to the farm household model.

2.4. Measure of resource heterogeneity

It would be helpful to have a measure of heterogeneity to predict which cooperatives have high increments in revenues. In general, from the theory of linear production games it can be shown that for farmers having identical ratios of resources potential increments in revenues when forming a coalition are zero. In that case farmers have similar excesses and deficits of resources, and will not mutually benefit from a cooperative agreement. We expect the increment to be higher when more complementarities are found in resources. Moreover, in Section 2.2.2, we assume that the transaction costs depend on the degree of heterogeneity in resources of farmers in a coalition. Hence, to conveniently compare the increments in net benefits for different coalitions, and to take the degree of heterogeneity into account in the transaction cost function, we define a measure of heterogeneity H_S for each coalition S. This measure is based on k resource ratios of each individual farmer i in a coalition, r_i^k.

First, we normalise the individual resource ratios of farmers r_i^k to $\tilde{r}_i^k$ by:

$$\tilde{r}_i^k = \frac{r_i^k - r_l^k}{r_h^k - r_l^k}. \quad (6)$$

Hereby $r_h^k = \max_{i \in N}\{r_i^k\}$ is the highest value of resource ratio k observed for all farmers and $r_l^k = \min_{i \in N}\{r_i^k\}$ the lowest value. This normalisation makes the ratios independent of the units of measure.

Secondly, based on these normalised values, we calculate the Euclidean distance:

$$D_{ij} = \sqrt{\sum_{k=1}^{K}\left(\tilde{r}_i^k - \tilde{r}_j^k\right)^2} \text{ for each } i,j \in S. \quad (7)$$

This describe the average difference D_{ij} between farmers i,j in coalition S. Hence, by Eq. (7), if all farmers in a coalition have similar ratios of resources, i.e., $\tilde{r}_i^k = \tilde{r}_j^k$ for all pairs of farmers i,j in coalition S, then they have zero distance, i.e., $D_{ij} = 0$ for all pairs i,j.

Finally, the measure of heterogeneity of a coalition S is defined by:

$$H_S = \frac{\sum_{i,j \in S, i<j} D_{ij}}{|S|} \text{ for each } S \subseteq N. \quad (8)$$

This is the sum of distances between farmers in a coalition, divided by the number of farmers in coalition S. Hence by Eq. (8), if all farmers have similar resource bundles, the measure of heterogeneity of the coalition H_S equals zero.

Land and labour resources are the most important complementary resources in the illustrative model in Section 3. Furthermore, both these factors are most frequently used in smallholder literature to describe heterogeneity in resources in crop production. Therefore, in Eqs. (6) and (7) we consider r_i^1 (and $\tilde{r}_i^1$): a (normalised) labour to upland ratio and r_i^2 (and $\tilde{r}_i^2$): a (normalised) labour to fadama land ratio.

3. An empirical example

In this section, we give an empirical illustration of the developed framework. The cooperative model is implemented and illustrated with a case study from northern Nigeria. First, we provide some general information on the data used, while the full formulation of the LP model is given in Appendix A. Secondly, we show the measure of heterogeneity for all coalitions. Finally, the cooperative game is applied to three different scenarios and the results are described.

3.1. Data

Agriculture in Nigeria employs about 45% of the total labour force in Nigeria, while it accounts for 40% of total GDP (Manyong et al., 2005). The case study is based on data collected in Ikuzeh village, Kajuru Local Government Area (Kaduna State, Nigeria), in the Northern Guinea Savannah. This agro-ecological zone is defined by a length of growing period of 151–180 days with a unimodal rainfall pattern. Kadara is the major ethnic group in the village. Main crops include sorghum, maize and cowpea for upland fields. In the lowland, or fadama, fields sugarcane and rice are cultivated during the raining seasons only. In this village, these fields are not commonly used for crop production outside the raining season, probably because the population density is still low. The main weekly market is distant. In 2002, a baseline survey was carried out in 39 randomly selected households. From this survey information is obtained on land use strategies, yields, input use, farm sizes and social characteristics like household size, education level, age of the household head and asset and livestock ownership. During the growing season of 2005, we collected additional data on a bi-weekly basis. This includes budget and market data as well as data on labour requirements for different crops. Furthermore, additional information on the wage labour market was collected during several field visits in 2006. The 2005 and 2006 price data was corrected for inflation, such that the set of prices is representative for 2002. In Appendix B, we present the village level data, which we use to determine the parameters of the objective function and the production matrix.

The baseline survey of 2002 is used to characterise the 39 households and estimate their resource parameters. Before we apply the model, for both computational and notational convenience, we use cluster analysis to identify the main groups of farmers, relatively homogenous in resources (Hazell and Norton, 1986). The 39 households are clustered using data on farm size, area of fadama fields owned, household size, livestock ownership and household stated assets, such as tools, radio, bicycle etc.

Based on the results of the hierarchical cluster analysis we group farmers into five clusters A, B, C, D, and E and calculate the average characteristics in each cluster (Table 3). In the remainder of this paper we refer to farmer A, B, C, D, and E as the average farmers based on the defined clusters in Table 3. Our model is applied to these five average farmers, representative for the observed resource heterogeneity in the village.

From Table 3, we learn that the available (outgoing) labour and the protein and energy requirements are strongly correlated with the household size. This is not surprising as those parameters depend on the composition of the households. Furthermore, we see that cluster A contains the largest group of farmers. The farmers in this cluster are the least endowed, since they have the smallest

Table 3
Characteristics of main clusters (farm typologies).

Cluster	*A*	*B*	*C*	*D*	*E*
Number of farmers	23	8	4	2	2
Farm size (ha)[a,b]	4.72	6.45	18.07	13.70	6.05
Fadama size (ha)[a,b]	0.45	0.72	1.96	2.23	0.52
Household size (number of persons)[a]	6.2	14.6	11.5	11.0	6.0
Ownership livestock (TLU)[a]	0.5	1.7	0.9	5.7	3.2
Value of stated assets (Naira)[a]	2900	5700	3000	2600	53,700
Labour (man hours/month)[b]	570	980	885	1104	561
Energy required (MJ)[b]	1484	2402	2778	2957	1560
Protein required (g)[b]	5664	9189	10,428	11,235	5898
Maximum loan (Naira)[b]	0	2850	2850	5700	5700

Source: Baseline survey, result from cluster analysis, 1USD = 133 Naira (December 2002).
[a] Cluster variable.
[b] Resource parameter.

Table 4
Measure of heterogeneity in resources in all possible coalitions.

S	H_S	*S*	H_S	*S*	H_S	*S*	H_S
AB	0.16	*ABC*	0.96	*ABCD*	1.32	N	1.55
AC	0.57	*ABD*	0.81	*ABCE*	1.17		
AD	0.47	*ABE*	0.44	*ABDE*	1.02		
AE	0.17	*ACD*	0.79	*ACDE*	1.05		
BC	0.71	*ACE*	0.76	*BCDE*	1.25		
BD	0.59	*ADE*	0.64				
BE	0.33	*BCD*	0.97				
CD	0.16	*BCE*	0.96				
CE	0.41	*BDE*	0.83				
DE	0.33	*CDE*	0.59				

For each possible coalition *S*, which consists of a subset of the average farmers *A*, *B*, *C*, *D*, and *E* as presented in Table 3, the measure of heterogeneity H_S in resources is calculated by applying the procedure described in Section 2.4. More homogenous coalitions display measures H_S closer to zero.

land and livestock holdings. Farmers from cluster *E* do not differ much from *A* with regards to household size and farm size, but both livestock ownership and stated assets are higher. Farmers from cluster *B* have similar farm size available as those from *E*, but the household size is larger and the stated assets and livestock units are smaller. Farmers from cluster *C* and *D* have both large farms and a larger than average household size, while those from cluster *C* have the largest farm size and those from cluster *D* are most endowed with livestock.

3.2. Measure of resource heterogeneity

In this section, we determine the degree of resource heterogeneity for all coalitions, as defined in Section 2.4. Table 4 shows the measures of heterogeneity H_S (see Eq. (8)), for all coalitions with $|S| > 1$, since by construction, H_S is zero for one-player coalitions. For example, considering coalition $\{A,B\}$, the normalised labour to land ratio and the normalised labour to fadama ratio (see Eq. (6)) for farmer *A* and *B* are: $\tilde{r}^1_A = 0.697$ and $\tilde{r}^1_B = 1$ are $\tilde{r}^2_A = 0.896$ and $\tilde{r}^2_B = 1$, respectively. Note that both ratios are 1 for farmer B, implying that his ratio of labour to both land types is highest. Applying Eqs. (7) and (8), respectively results in $H_{\{A,B\}} = 0.16$. H_S is calculated for the other coalitions in a similar way.

Table 4 shows that of all two-player coalitions, $\{A,B\}$ and $\{C,D\}$ have the lowest measure of heterogeneity, while coalition $\{B,C\}$ has the highest value. This indicates that resource ratios are similar for farmers *A* and *B*, and farmers *C* and *D*, while they are different for farmers *B* and *C*. This is also reflected in the three-player coalitions, where we see that the coalitions in which farmers *B* and *C* are involved, have a high measure of heterogeneity. Moreover, the three-player coalition $\{A,B,E\}$ has a low heterogeneity measure.

3.3. Cooperative games

In this section, we consider three different scenarios, representing three different assumptions on the transaction cost function. First, we assume transaction costs to be zero (Scenario 1), providing a benchmark on the gains of cooperation. Next, we assume the transaction cost function $tc(s,a,h)$ to be a linear function (Scenario 2), and finally, we assume an exponential function (Scenario 3). We denote the resulting games for the three scenarios by (N,v_1), (N,v_2), and (N,v_3) respectively. For each scenario the respective games and the relation between the increments in net benefits of cooperation and the degree of heterogeneity are described. Finally, we apply the different solution concepts to each game and compare the results.

3.3.1. Scenario 1: Game (N,v_1) with $tc(s,a,h) = 0$

In this scenario, we assume that the transaction costs are zero for each coalition. By consequence, the cooperative game (N,v_1) equals the linear production game for revenues (N,r). Hence, we apply the linear production game to the five average farmers $N = \{A,B,C,D,E\}$ (Table 3), and construct the game as defined in Eq. (3), using the LP model from Appendix A.

This results in an optimal farm plan for each coalition. We expect that the farm plan of a coalition differs from the summed individual farm plans. This is confirmed when comparing the optimal farm plans of the individuals and the farm plan of the grand coalition (Fig. 1). For ease of comparison, we sum the individual farm plans (i.e., the case without cooperation) in Fig. 1 as well.

First, observe that all individual farm plans are different, which is clearly a result of the different resources. Note that the fallow land of farmers *C* and *D* is due to their large land holdings (see also Table 1) and the lack of complementary resources. Further, in all farm plans Sorghum–Cowpea relay is the dominant cropping system, with more than 50 % of the cultivated area allocated to these crops.

When comparing the sum of the individual farm plans with the farm plan of the cooperative, it shows that all individual farmers together fallow approximately 12 hectares, while in the cooperative fallow land reduces to 8.5 hectares. This is likely a combination of the fact that individual farmers do not have sufficient access to capital to hire more labour, and/or the return to labour is too low. Area used for growing Sorghum–Cowpea relay and sugarcane is expanded, while cultivation of okra, cassava and hungry rice decrease in cooperation. Note that this shift is towards sugarcane, a high-value crop, and Sorghum–Cowpea, both products with high nutritional values. This is a critical feature of the cooperative model, whereby the access to aggregated individual resources, leads to an adjustment of the cooperative farm plan, or scaled individual farm plans, and areas devoted to crops.

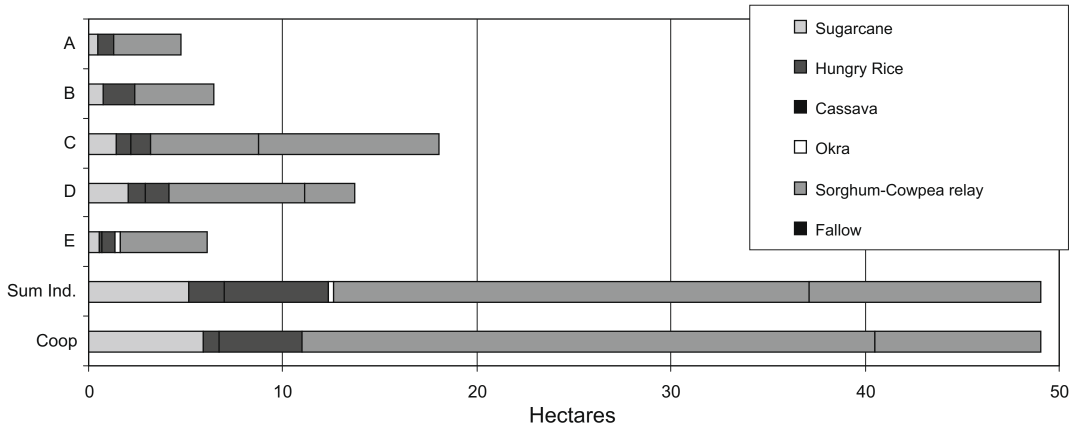

Fig. 1. Optimal farm plans of individual farmers and the grand coalition. (The figure shows the composition of optimal farm plans for individual average farmers A, B, C, D, and E, the sum of individual farmers and the grand coalition).

Table 5
Net benefits of coalitions without transaction costs (Game (N,v_1)).

S	$v_1(S)$	S	$v_1(S)$	S	$v_1(S)$	S	$v_1(S)$	S	$v_1(S)$
A	271,099	*AB*	667,173	*ABC*	1,507,522	*ABCD*	2,351,735	*N*	2,685,004
B	396,074	*AC*	954,334	*ABD*	1,625,392	*ABCE*	1,835,246		
C	604,609	*AD*	1,137,887	*ABE*	999,496	*ABDE*	1,955,472		
D	800,591	*AE*	600,965	*ACD*	1,754,925	*ACDE*	2,166,980		
E	321,885	*BC*	1,212,479	*ACE*	1,328,126	*BCDE*	2,388,637		
		BD	1,349,520	*ADE*	1,471,518				
		BE	728,397	*BCD*	2,050,770				
		CD	1,405,200	*BCE*	1,541,344				
		CE	1,029,093	*BDE*	1,679,716				
		DE	1,173,810	*CDE*	1,866,015				

Columns *S* indicate the composition of coalitions, whereby *A*, *B*, *C*, *D*, and *E* represent the average farmers as in Table 3. The columns $v_1(S)$ indicate the value (net benefits) of the coalition. Values are given in Naira, 1USD = 133 Naira (December 2002).

Moreover, these adjustments lead to increased overall value. Table 5 presents the complete game (N,v_1). Hence, $v_1(S)$ reflects the total value of coalition *S*, which equals the gross margin resulting from the LP model solved for coalition *S*.

Table 5 shows that the sum of the gross margin of all individual farmers $(A + B + C + D + E)$ is equal to 2,394,258 Naira, while the gross margin of the cooperative is 2,685,004 Naira, an increase of 12%. Hence, the increment in net benefits (Eq. (2)) for the grand coalition, $\Delta(N) = 290{,}746$.

Recall from Table 3 that farmers *C* and *D* are both well endowed, reflected in their relatively high gross margins (value) in Table 5, while coalitions including these farmers display higher gross margins as well. However, their resources are not complementary, which is shown by a low measure of heterogeneity H_S in Table 4, they have similar resource deficits and shadow prices and cooperation does not lead to extra gains. This is reflected in Table 5 as $v_1(\{C,D\}) = v_1(\{C\}) + v_1(\{D\})$. Observe that the same argument, although for different resources, holds for the farmers *A* and *B*, which both have excess of labour and shortage of land. This finding similarly holds for other coalitions composed of farmers with non-complementary resources.

On the other hand, farmers *A*, *B* and *E* have similar excess resources, hence similar resource ratios, as well as farmers *C* and *D* for different resources (see the measure of heterogeneity H_S in Table 4). Intuitively, coalitions formed by two or more players from both these groups, exploiting complementarities, give extra gains to the players. For example, $v_1(\{B,C\})$ is considerably higher than the sum of the individual values $v_1(\{B\})$ and $v_1(\{C\})$. In fact the increment of a coalition of two players is highest for this coalition.

Fig. 2 confirms that there is a positive relation between the degree of heterogeneity and the increment in net benefits of a coalition. As illustrated by the plotted regression line, the increment in net benefits is higher for coalitions with a higher degree of heterogeneity in resources. A linear regression has a fit of (R^2) 0.84 and the slope coefficient is significantly greater than zero $(p < 0.001)$. The 95% confidence interval for the slope coefficient is (188,116; 272,196).

Summarizing, the results show, as expected, that many coalitions make more efficient use of farm resources than farms acting alone. Hence, farmers engaging in a cooperative agreement are likely to adjust their farm plan, i.e., crop sizes are endogenous, based on access to an aggregated pool of complementary resources. This is different from the results of the single-crop model in Gilligan (2002), where farm sizes were considered independent from membership of a labour team.

Moreover, the results show that coalitions of farmers with heterogeneous resources benefit most from cooperation. These results confirm common intuition whereby, for example, cooperation between two farmers is most beneficial when one has excess of one resource, being in deficit for the other, and vice versa for a different resource. To the contrary however, a body of empirical research (e.g. Thorp et al., 2005; Bernard et al., 2008a, 2008b) shows that most cooperatives are being formed in either relatively homogeneous villages or between groups of homogeneous farmers. This contradiction suggests transaction costs in cooperatives play an important role.

3.3.2. Scenario 2: Game (N,v_2) with $tc_1(s,a,h)$ linear function

As discussed above, the cooperative game assuming zero transaction costs provides an upper bound on the gains from cooperation. In fact this upper bound equals $\Delta(N) = 290{,}746$ Naira in the grand coalition (Table 5), which on average translates to approxi-

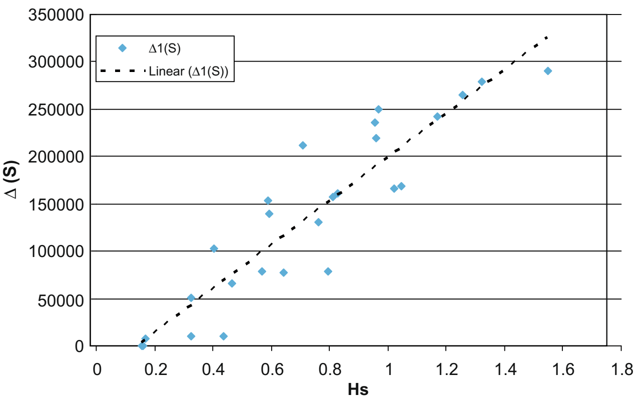

Fig. 2. Increments in net benefits of coalitions in relation to resource heterogeneity in coalitions (H_S) for the case that no transaction costs are included ($\Delta_1(S)$).

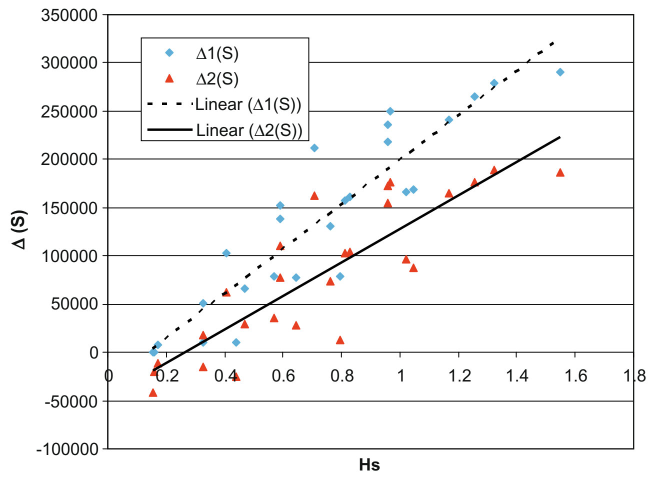

Fig. 3. Increments in net benefits of coalitions in relation to resource heterogeneity in coalitions (H_S) for the case no transaction costs are included ($\Delta_1(S)$) and the case transaction costs are linear ($\Delta_2(S)$).

mately 6000 Naira per household member per year (which is based on a total of 49 members in the five average households). This increment is substantial and compared with the daily wage rate (156 Naira), this increment equals approximately 38 working days per member. By consequence the transaction costs of cooperation should be substantial as well, in order to make cooperation an unattractive practice. In this section we illustrate the effect on the expected net benefits of a cooperative of a transaction cost function linear in its arguments. Note that the farm plan in this scenario does not change compared to Scenario 1, since the revenue game (N,r) does not change. Following the discussion in Section 2.3, we assume that $c(S) = tc_1(s,a,h)$ is a linear function (Eq. (4)) of the number of households, the total cropping area and the measure of heterogeneity (for $|S| > 1$).

As we mentioned above, when the transaction costs are higher than 290,746 Naira, or 38 working days per member, cooperation becomes unattractive. We illustrate a case whereby the parameters in the transaction cost function are likely to be realistic. We assume that each household spends 4 days on meetings before the season and during the cropping season meetings are held twice a month for one afternoon $(4*6*26 + 6*2*4*26 = 12*6*26)$ $\alpha_1 = 1872$. This equals 60 days in the grand coalition. Furthermore, we assume six extra hours per month per hectare are included for travelling costs, which equals $\beta_1 = 6*6*26 = 936$ during the 6 months cropping season. This is equal to approximately 330 days in the grand coalition. Moreover, we assume that for each unit increase in the degree of heterogeneity, one additional person is needed daily during the cropping season to monitor and reduce moral hazard and free ridership, hence $\chi_1 = 6*30*6*26 = 28{,}080$. This is equal to approximately 280 working days to reduce moral hazard in the grand coalition. Hence, in this case the total transaction costs of the grand coalition equal approximately 670 days. With these parameters the grand coalition has an increment of net benefits of 186,506 Naira. Fig. 3 illustrates the increments of net benefits for all coalitions for both $v_1(S)$ (without transaction cost) and $v_2(S)$ (after including linear transaction costs) as well as regression lines relating heterogeneity to increments in net benefits.

As illustrated by the plotted regression lines, the difference in potential increments in net benefits is larger for coalitions with a higher degree of heterogeneity in resources. Hence, coalitions with a higher degree of heterogeneity (which are often the coalitions with more players and with more available land) have higher transaction costs. Nevertheless, the change in net benefits is negative for some smaller coalitions when including transaction costs. If the formation of cooperatives is a stepwise process, starting from smaller coalitions, this may prevent initial formation taking place or being successful, even though gains for larger coalitions are positive.

Next, we calculate how much the transaction costs can increase such that cooperation in the grand coalition becomes unattractive. For this to happen, the total transaction costs of the grand coalition should increase by 179%. Now, we analyse the different components of the transaction cost function, and calculate how much one of the parameters can increase such that the total gain in the grand coalition equals its transaction costs. This analysis shows that, ceteris paribus, α_1 can increase 2090% before cooperation becomes unattractive for the grand coalition. Similarly, ceteris paribus, β_1 and χ_1 can increase by 463% and 528% respectively.

3.3.3. Scenario 3: Game (N,v_3) with $tc_2(s,a,h)$ exponential function

Intuitively, when coalitions consist of many households, take decisions on larger areas and farmers are more heterogeneous, it takes longer to agree on decisions. Moreover, these costs are likely to grow exponentially instead of linearly. Therefore, we next consider an exponential transaction cost function (Eq. (5)).

In the game with an exponential cost function, we assume the parameters $\alpha_2 = 1872$, as in Scenario 2, $\beta_2 = 2$, reflecting a quadratic increase in travel costs, and $\chi_2 = 10$, reflecting strongly increasing costs of moral hazard and free ridership, distrust and risk of default in more heterogeneous coalitions. For the grand coalition these parameters reflect meeting costs of 300 working days, and other transaction costs of 1544 working days. In this case the increment in net benefits in the grand coalition is 3070 Naira. However, minor increases in transaction costs will lead to a net loss in the grand coalition. Fig. 4 illustrates the increments in net benefits of the coalitions for both $v_1(S)$ and $v_3(S)$. As well regression lines for the increments $\Delta_1(S)$ and $\Delta_3(S)$ for the respective games (N,v_1) and (N,v_3) are plotted.

We see that the gains of cooperation in the third scenario (Δ_3) are highest for coalitions in which the degree of heterogeneity is about 1, resulting from the assumption on the shape of the transaction cost function. These coalitions are of size 3 or 4 (see Table 4), while even the increment in net benefits in the grand coalition remains a little positive. The main finding that emerges from Fig. 4 is that for exponentially increasing transaction costs the optimal size of the coalition declines, and the largest and most heterogeneous coalition is not always most profitable. Hence, the grand coalition is not formed. This finding could explain some of the empirical observations (e.g. World Bank, 2008b; Bernard et al., 2008a)

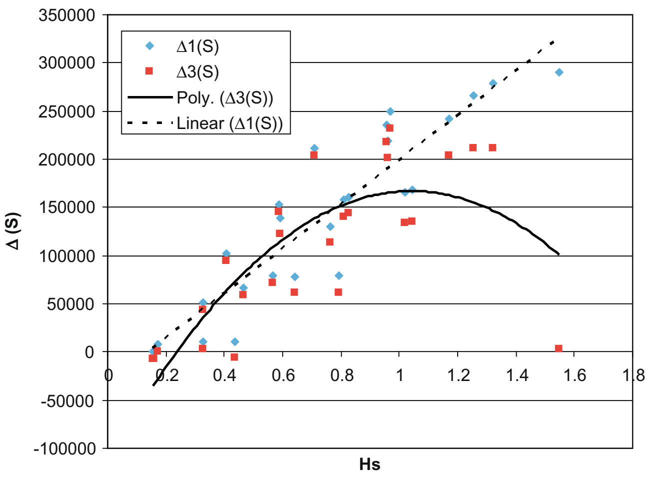

Fig. 4. Increments in net benefits of coalitions in relation to resource heterogeneity in coalitions (H_S) for the case no transaction costs are included ($\Delta_1(S)$) and the case transaction costs are exponential ($\Delta_3(S)$).

Clearly, the shape of the transaction cost function plays an important role in finding the optimal size and composition of coalitions (cooperatives). Furthermore, some small relatively homogenous coalitions face negative returns. If the development of cooperatives is a stepwise process, initially starting with small groups, this may actually impede cooperatives being formed at all, despite positive returns in some larger coalitions.

3.4. Game theoretical solution concepts

While the exact shape of the transaction costs function is clearly important in the formation of optimal cooperatives, the main findings from Figs. 2–4 make it clear that in many cases cooperation leads to substantial improvements. However, to actually form the cooperative, each individual farmer needs to have a clear economic incentive to join. Potentially, one of the main impediments to engage in cooperative agreements is distrust between farmers in general (e.g. Tu and Bulte, 2007) and/or distrust about the fairness of the division rule (Cruijssen et al., 2005). Therefore solution concepts from cooperative game theory are used to determine fair division rules, taking into account each player's contribution to the cooperative.

In Table 6, we show the results for the Shapley value (see Section 2.3). The first row displays the individual gross margins, which a farmer can attain by operating individually. The next rows show the allocation of net benefits in the grand coalition based on the Shapley value for the three scenarios, and the relative increases of net benefits compared to the individual gross margins.

Recall from Table 4 that a coalition containing farmers *B* and *C* is relatively heterogeneous, which results in relatively large increases for both of them in Table 6. Furthermore, we observe that all allocations in Scenario 1 and 2 are individually rational, i.e., each allocation to each player is higher than the individual gross margin. For example, in Scenario 1, farmer *A* can earn 293,549 Naira by cooperating with the other farmers, an increase of 22,450 Naira. However, the negative relative increases for farmers *A*, *D*, and *E* in Scenario 3 indicate that this is not an individual rational allocation.

This means that farmers *A*, *D*, and *E* are worse off by joining the grand coalition, which confirms the finding in Scenario 3, that forming smaller coalitions is more attractive. Therefore, next we explore if sub-coalitions can be formed. We take for example one coalition of farmers *B* and *C*, $N_1 = \{B,C\}$, and another coalition with farmers *A*, *D*, and *E*, $N_2 = \{A,D,E\}$, and consider the two corresponding subgames (N_1, v_3) and (N_2, v_3). For both subgames the Shapley value is calculated, as well as the relative increase for every farmer (Table 7).

Table 7 shows that the allocations in both sub-coalitions are individually rational. Hence, the formation of smaller coalitions is more attractive than either operating alone or forming the grand coalition. The results show that cooperation is attractive to all farmers, while in some cases, depending on the transaction costs, it is possible to form a grand coalition, and in other cases it is more attractive to form sub-coalitions.

Table 6
Calculating the relative increase in net benefits when all farmers join the grand coalition based on Shapley value.

	A	*B*	*C*	*D*	*E*
Individual gross margin	271,099	396,074	604,609	800,591	321,885
S.1: no transaction costs					
Shapley value for (N,v_1)	293,549	489,746	702,090	853,221	346,399
Relative increase (%)	8	24	16	7	8
S.2: linear transaction costs					
Shapley value for (N,v_2)	278,430	469,480	672,330	829,010	331,520
Relative increase (%)	3	19	11	4	3
S.3: exponential transaction costs					
Shapley value for (N,v_3)	238,340	429,290	641,110	793,780	294,800
Relative increase (%)	−12	8	6	−1	−8

In this table, the individual gross margins of average farmers *A*, *B*, *C*, *D*, and *E* when farming alone, are compared with the allocations in the grand coalition for three transaction cost scenarios. Values are given in Naira, 1USD = 133 Naira (December 2002).

Table 7
Calculating the relative increase in net benefits when farmers join a partial cooperative in Scenario 3 based on Shapley value.

	A	*B*	*C*	*D*	*E*
Individual gross margin	271,099	396,074	604,609	800,591	321,885
Shapley value for (N_1,v_3)	–	4,98,216	7,06,751	–	–
Shapley value for (N_2,v_3)	286,713	–	–	837,881	330,067
Relative increase for N_1	–	26%	17%	–	–
Relative increase for N_2	6%	–	–	5%	3%

In this table, the individual gross margins of average farmers *A*, *B*, *C*, *D*, and *E* when farming alone, are compared with the allocations in two sub-coalitions ($N_1 = \{B,C\}$ and $N_2 = \{A,D,E\}$) in Scenario 3. Values are given in Naira, 1USD = 133 Naira (December 2002).

The heterogeneity of the resources of farmers in a coalition plays an important role in revenues and costs of cooperation, while at the same time division of net benefits in heterogeneous cooperatives is more complex. The Shapley value can be used to divide net benefits in a fair way. Using such a fair division rule could facilitate the development of more diverse cooperatives, in which increments in net benefits remain positive in many cases. Further research should determine how such division rules can be implemented in a rural smallholder setting.

4. Conclusion

While cooperative agreements in production continue to play an important role for many smallholder farmers in developing countries, the costs and benefits of cooperation are rarely quantified. Moreover, most research on production cooperatives is based on single-crop models and/or focuses on one resource. To fill this gap, we have introduced a framework to analyse costs and benefits in smallholder farm cooperatives. This approach combines farm household models with cooperative game theory to develop a cooperative farm household game. To estimate the revenues and costs of coalitions, i.e., cooperatives consisting of various farmers, we developed a cooperative revenue game and a cooperative cost game. The revenue game is a linear production game based on a linear farm household model. The cost game is based on a transaction cost function, which depends on the number of households in the coalition, the total cropping area of the coalition and the heterogeneity in resources of the coalition. Finally, the net benefits of each coalition, i.e., the assigned values in the farm household game, equal the revenue minus the transaction costs in a coalition.

We have illustrated the cooperative farm household model with an empirical example from northern Nigeria. Three different scenarios are considered, based on three different shapes of the transaction cost function. These are cases in which transaction costs are zero, a linear or an exponential function, respectively. The first scenario describes an upper bound or benchmark for the maximal net benefits of cooperation, while the second and third scenarios show that in some cases transaction costs render cooperation unattractive.

The illustration of the framework can primarily be viewed as an extension of the commonly found practice of labour exchange. The cooperative farm plan presented in this paper can be proportionally implemented at each individual household, such that farmers remain managers on their own fields. The primary cooperation during the cropping season is based on non-proportional exchange of labour during production and of products after harvest. The set-up of such cooperation has some further benefits, not explicitly considered in this paper, such as the marketing of products, the purchase of inputs, as well as the possible economies of scale in using labour teams in the production (e.g. Gilligan, 2002).

On the other hand, certain other factors, such as trust, power and social norms have been ignored. Trust between different households, or the lack of it, might still be covered through some of the transaction costs and/or the use of fair division rules. Furthermore, inequality in power relations between rich and poor, or older and younger farmers may impede the formation of cooperatives. Moreover, social norms play an important role. Bernard et al. (2008b) note that cooperatives are mostly found in homogenous villages, whereby homogeneity might be a result of social norms which dictate that farmers assist each other. Clearly, variation in transaction costs across locations is likely to depend on such norms.

Furthermore, in our framework we assumed farmers to be homogeneous with respect to their utility function, as well as market participation. Since richer farmers might be better integrated into markets, they are likely to benefit less from cooperation. Similarly, different production objectives, as well as different levels of risk aversion could further limit the potentials of cooperation.

These observations should be addressed in further empirical and theoretical research. While theoretical research should focus on including heterogeneity in utility functions and market integration, further empirical research is needed on the formation of cooperatives. Bernard et al. (2008a), for example, shed some light on the types of farmers benefiting from cooperatives, but do not provide estimates of the magnitudes and differences of such benefits to participants. Additional empirical research should focus on estimating the magnitudes of costs incurred and benefits accrued, how these are divided amongst members of cooperatives, and how policies can contribute to improving the functioning of such systems.

The quantitative framework presented in this paper is a first step towards unravelling the mechanism of cooperation in rural production. The framework can be viewed as a starting point for studying different types of cooperation, considering linear or non-linear production processes, in different settings and regions.

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Appendix A

Objective :

$$\begin{aligned}\text{GrossMargin} = &\sum_{i,j,t} \text{AREA}_j * \text{yield}_{ijt} * \text{cropprice}_t \\ &- \sum_t \text{LABOURHIRED}_t * \text{wagerate}_t \\ &- \sum_{j,t} AREA_j * \text{fertilizerrequired}_{j,t} * \text{fertilizerprice}_t\end{aligned}$$

Restrictions

$$\sum_j \text{AREA}_j \leqslant \text{landavailable} \tag{A.1}$$

$$\sum_{j \in C_{\text{fadama}}} \text{AREA}_j \leqslant \text{fadamaavailable} \tag{A.2}$$

$$\begin{aligned}&\sum_j AREA_J * \text{labourrequired}_{j,t} \\ &\quad \leqslant \text{labouravailable} + \text{LABOURHIRED}_t - \text{LABOUROUTHIRED}_t \\ &\qquad \forall t = 1, \ldots, T\end{aligned} \tag{A.3}$$

$$\begin{aligned}&\text{STORE}_{i,t} = \text{STORE}_{i,t-1} + \sum_j \text{AREA}_j * \text{yield}_{ijt} - \text{CONSTORE}_{i,t} - \text{SELL}_{i,t} \\ &\quad \forall i = 1, \ldots, K, \quad \forall t = 1, \ldots, T\end{aligned} \tag{A.4}$$

$$\begin{aligned}&\text{FOODSTORE}_{i,t} = \text{FOODSTORE}_{i,t-1} + \text{BUY}_{i,t} - \text{CONFOODSTORE}_{i,t} \\ &\quad \forall i = 1, \ldots, K, \quad \forall t = 1, \ldots, T\end{aligned} \tag{A.5}$$

$$\text{STORE}_{i,t} + \text{FOODSTORE}_{i,t} = 0 \quad \text{for} \quad i \in K_{\text{perishable}}, \quad \forall t = 1,\ldots,T \tag{A.6}$$

$$\text{STORE}_{i,0} = \text{initialstore}_i \quad \forall i = 1,\ldots,K \tag{A.7}$$

$$\text{FOODSTORE}_{i,0} = \text{initialfoodstore}_i \quad \forall i = 1,\ldots,K \tag{A.8}$$

$$\begin{aligned} CAPITAL_t = {} & CAPITAL_{t-1} + NEWLOAN_t - REPAYLOAN_t \\ & + \sum_i (SELL_{i,t} - BUY_{i,t}) * cropprice_{i,t} \\ & - \sum_j AREA_j * fertiliserrequired_{i,j} * fertiliserprice \\ & + nonagriculturalincome_t - otherexpenses_t \\ & + (LABOUROUTHIRED_t - LABOURHIRED_t) * wagerate_t \\ \forall t = 1,\ldots,T & \end{aligned} \tag{A.9}$$

$$\text{CAPITAL}_0 = \text{initialcapital} \tag{A.10}$$

$$\sum_i (\text{CONSTORE}_{it} + \text{CONFOODSTORE}_{it}) * \text{cropnutrients}_{ih} \leqslant \text{nutrientequired}_h \quad \forall T = 8,\ldots,T, \quad \forall h = 1,\ldots,H \tag{A.11}$$

$$\sum_i (\text{STORE}_{iT} + \text{FOODSTORE}_{iT}) * \text{cropnutrients}_{ih}/7 \geqslant \text{nutrientrequired}_h \tag{A.12}$$

$$\sum_i \text{REPAYLOAN}_t = \sum_t \text{NEWLOAN}_t \tag{A.13}$$

$$\sum_t \text{NEWLOAN}_t \leqslant \text{maximumloan} \tag{A.14}$$

$$\text{LABOUROUTHIRED}_t \leqslant \text{outgoinglabour} \tag{A.15}$$

$$\text{Alldecisionvariables} \geqslant 0 \tag{A.16}$$

Appendix B. Village characteristics and data used in the model

Table B1.

The other data in the production matrix were the wage rates, which were estimated based on the fortnightly surveys of 2005 and fertiliser prices obtained from the Kaduna State Fertilizer Company. The average wage rate was approximately 26 Naira per hour and the average fertiliser price was 37 Naira per kg.

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Table B1
Crops and cropping systems included in the model.

Cropping systems	Crops	Total required hours	Fertiliser required (kg)	Yields (kg/hectare)	Average crop price (Naira/kg)	Energy (MJ/kg)	Protein (g/kg)
Sugarcane (sole) (low input)	Sugarcane	802	274	3720	46	0.251	10
Rice (sole) (low input)	Rice	1421	13	775	59	1.508	75
Rice (sole) (high input)	Rice	1485	85	1107	59	1.508	75
Maize (sole) (low input)	Maize	593	27	327	32	1.492	95
Maize (sole) (High Input)	Maize	640	122	750	32	1.492	95
Sorghum (sole) (low input)	Sorghum	487	0	200	31	1.437	101
Sorghum (sole) (High Input)	Sorghum	571	67	400	31	1.437	101
Sorghum–Cowpea relay	Sorghum	617	0	455	31	1.437	101
(low input)	Cowpea		0	145	61	1.446	222
Late Millet (sole) (low input)	Late Millet	487	8	637	33	1.425	97
Hungry Rice (sole) (low input)	Hungry rice	253	0	335	69	1.399	122
Soybean (sole) (low input)	Soybean	549	0	363	32	1.404	380
Groundnut (sole) (low input)	Groundnut	434	0	406	27	1.626	182
Cassava (sole) (low input)	Cassava	977	0	1682	17	0.457	9
Cocoyam (sole) (low input)	Cocoyam	706	0	1158	24	0.360	15
Okra (sole) (low input)	Okra	253	0	400	68	0.130	16
Hot Pepper (sole) (low input)	Hot pepper	253	9	435	40	0.117	12

Source: Data collection 2002 average crop price obtained from Kaduna state development program, and the energy and protein contents (FAO, 2007) of each crop.

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